Exercise II, Theory of Computation 2025

These exercises are for your own benefit. Feel free to collaborate and share your answers with other students. Solve as many problems as you can and ask for help if you get stuck for too long. Problems marked * are more difficult but also more fun :).

These problems are taken from various sources at EPFL and on the Internet, too numerous to cite individually.

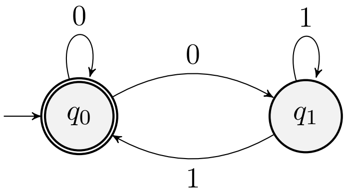
1 For both of the following languages over $\Sigma = \{0, 1\}$, construct an NFA which recognizes it.

1a $\{w \mid w$ has a 1 in the third to last position$\}$

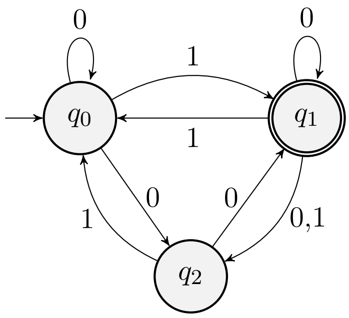
1b $\{w \mid w$ contains four consecutive symbols that are the same$\}$

2 For both of the NFA over $\Sigma = \{0, 1\}$ below, construct a DFA that accepts the same language. First, write down the subset construction and then remove any states that are not reachable.

2a



2b



3 Using non-determinism, give an alternative proof of the fact that the union of two regular languages is regular. Given that we already know regular languages to be closed under complementation, explain how this proof directly extends to intersections.

4 Given an NFA over some alphabet Σ, show how to construct an NFA over Σ that accepts the same language, but does not contain any ε-transitions.

5 For a language L over some alphabet Σ, we define its *Kleene closure* as

$$L^* = \{w_1 w_2 \cdots w_k \mid k \geq 0 \text{ and } w_1, w_2, \ldots, w_k \in L\}.$$

In other words, L^* contains all strings obtainable by concatenating some (not necessarily distinct) strings from L. Note in particular that $\varepsilon \in L^*$ and $L \subset L^*$, corresponding to $k = 0, 1$.

Prove that if L is regular then L^* is also regular.

6* Given a language L over $\Sigma = \{0, 1\}$, define

$$L_{11} = \{xy \mid x11y \in L\}.$$

In other words, L_{11} is the language of all strings obtainable by taking some string in L and deleting two consecutive 1s from it.

Prove that if L is regular, then L_{11} is also regular.

Hint: *Add transitions that simulate skipping over two 1s. Ensure exactly one of them is used.*

7* For two strings $a, b \in \Sigma$ of equal length, their *Hamming distance*, denoted by $H(a, b)$, is the number of indices at which a and b differ. Given a language L over alphabet Σ, define

$$\Gamma(L) = \{w \mid \text{there is some } w' \in L \text{ with } |w'| = |w| \text{ and } H(w, w') \leq 1\}.$$

Prove that if L is regular, then $\Gamma(L)$ is also regular.

Hint: *Add transitions that simulate a wrong input symbol. Ensure at most one of them is used.*

8 In lectures, we have seen that if L is regular, then so is its complement $\overline{L}$. This was done by taking a DFA that recognises L and changing accepting states for non-accepting states and vice versa. Could we do the same with an NFA that recognises L instead? Explain your answer.